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About the Author

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PRECALCULUS

Chapter 1: Functions and Graphs

PDF Version

1.7: Parametric Relations

A parametric relation is a relation where both x and y are defined in terms of a third variable. This third variable is called a **Parameter.** A plane curve is defined by the set of points (x, y) such that $x = f(t)$ and $y = g(t)$ where f and g are both defined on an interval I. The equations $x = f(t)$ ane $y = g(t)$ are the **parametric equations** of the curve, with the **parameter** t.

EXAMPLE 1:

Consider the set of all ordered pairs $x = t + 1$ and $y = t^2$, where t is any real number. Find the points determined by $t = -2, \quad -1, \quad 0, \quad 1, \quad 2$. Also, find an algebraic relationship between the variables x and y, and graph this relation.

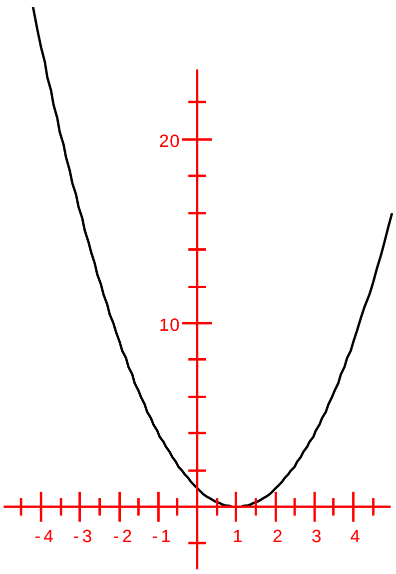
Solution:

t	$x = t + 1$	$y = t^2$	(x, y)
-2	-1	4	$(-1, 4)$
-1	0	1	$(-1, 1)$
0	1	9	$(0, 1)$
1	2	1	$(2, 1)$
2	3	4	$(2, 3)$

To find the relationship between x and y, we eliminate t between the two equations:

$$x = t + 1$$
$$t = x - 1$$
$$y = t^2$$
$$y = (x - 1)^2$$

Graph of this relationship is shown below:



Example 2:

Find coordinates of the point represented by $t = -1$ in the relationship $x = 3t - 5$ and $y = 6 - 2t$.

Solution:Substituting the given value of t in the two equations, we get

$$x = 3(-1) - 5$$
$$= -8$$
$$y = 6 - 2(-1)$$
$$= 8$$

Hence, the coordinates of the point is $(-8, 8)$.

Parametric equations for a given relationship is not unique. We can assume either x of y as a parameter either alone as an equation involving the parameter, and then substitute in the given relationship to get the corresponding parametric equation for y. A parameter is represented by any symbol of though commonly used parameters are t and θ.

EXAMPLE 3:

Find a pair of parametric equations for the relationship
$x^2 + y^2 = 4, \quad x, y \geq 0$

Solution: First, we solve the above equation for y:

$$y^2 = 4 - x^2$$
$$y = \sqrt{4 - x^2}$$

As per the given conditions, y is positive. Also, from the solution for y, we see that the restriction of x is $-4 \leq x \leq 4$
Now, let the parameter be t, and let $x = t$. So, we get the second parametric equation as $y = \sqrt{4 - t^2}$, with $-4 \leq t \leq 4$.
If we take the first equation as $x = t^2$, then, the second equation would be $y = \sqrt{4 - t}$, with $t \leq 4$.

Practice Problems

(1) For the parametric relationship $x = 3t$ and $y = t^2 - 4$, find the

$(6, 0)$

(2) For the parametric relationship $x = \sqrt{t + 1}$ and $y = 2t - 5$, fir

$(2, 1)$

(3) Find the rectangular equation for the parametric relationship x

$(y+5)^2 = 4x$

(4) For the parametric relationship $x = 2t$ and $y = 5t + 1$, find the

$(-4,-9),(-2,-4),(0,1),(2,6),(4,1)$

(5) Find the points determined by the values of $t = -2, -1, 0, 1, 2$

$(0,-3),(-1,-2),(0,-1),(3,0),(8,1)$

(6) Find a parametric relation for the function $y = 2x + 7$

If we take $x = t$, then, $y = 2t + 7$

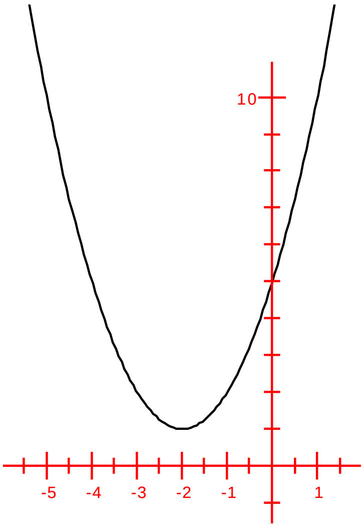
(7) Find a parametric relation for the function $x^2 + y^2 = 4$

$x = 2\cos\theta$ and $y = 2\sin\theta$

For problems #8-12, for each plane curve, find the rectangular form and then graph the curve:

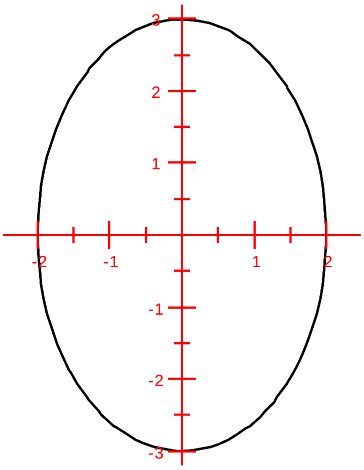
(8) $x = t + 2$ and $y = t^2 + 1$

$y = (x-2)^2 + 1$



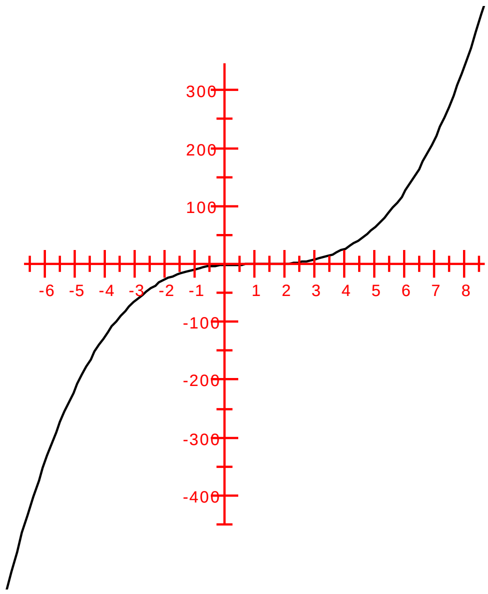
(9) $x = 2\sin t$ and $y = 3\cos t$

$\frac{x^2}{4} + \frac{y^2}{9} = 1.$



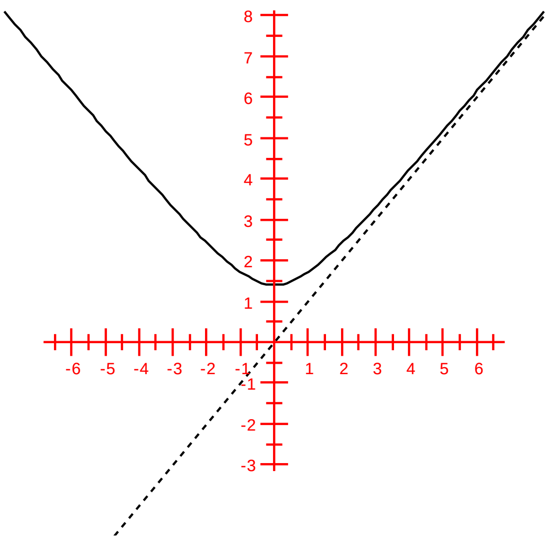
(10) $x = t^3 + 1$ and $y = t^3 - 1$

$y = (x-1)^3 - 1$



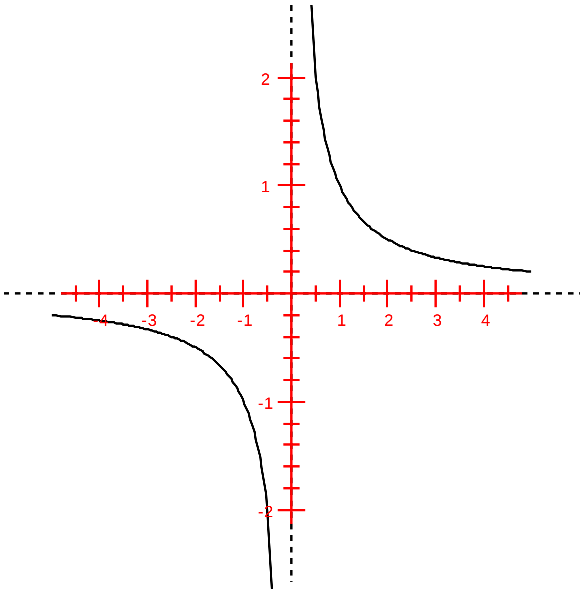
(11) $x = t$ and $y = \sqrt{t^2+2}$

$y = \sqrt{x^2+2}$

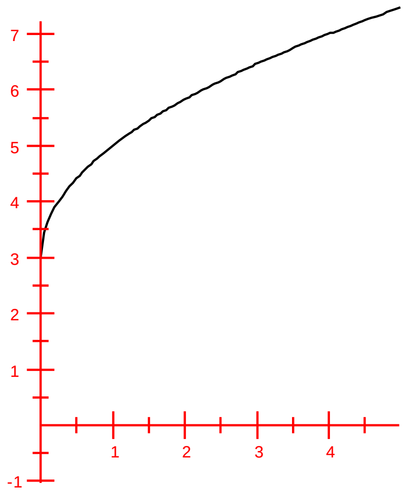


(12) $x = t+2$ and $y = \frac{1}{t+2}$

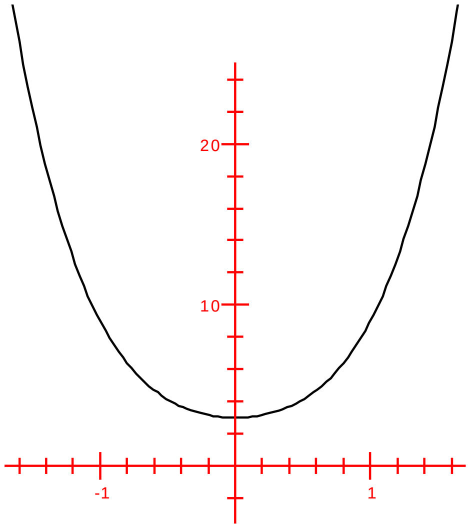
$y = \frac{1}{x}$



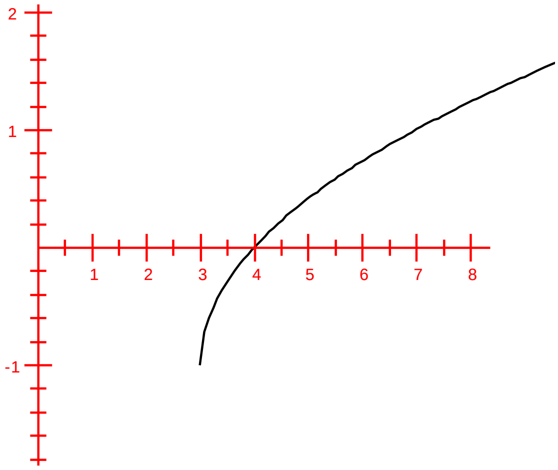
(13) Let $x = t^2$n and $y = 2t + 3$. Graph the plane curve defined b



(14)Graph the equation defined by the parametric relationship $x =$



(15) Graph the plane curve defined by the parametric equations x



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