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About the Author

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PRECALCULUS

Chapter 1 - Functions and Graphs

PDF Version

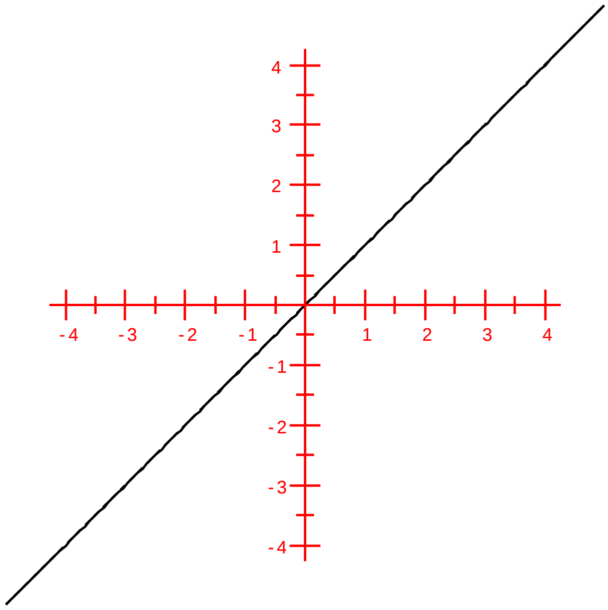
1.4: Twelve Basic Functions

In this section, we discuss 12 of the several basic functions, based on which other functions can be graphed using a series of transformations. Most of these are available in the graphing calculator. Analyzing and understanding the properties of these twelve basic functions help us to analyze and understand behavior and properties more complex functions. This is an essential skill to be developed before we embark on the study of Calculus. Below is the list of the twelve basic functions we shall discuss in this section.

1. The Identity Function
2. The Square Function
3. The Cubic Function
4. The Reciprocal Function
5. The Square Root Function
6. The Exponential Function
7. The Natural Logarithm Function
8. The Sine Function
9. The Cosine Function
10. The Absolute Value Function
11. The Greatest Integer Function
12. The Logistic Function

The Identity Function:

Algebraic notation of this funciton is $f(x) = x$. This function maps every real number to itself. This shape is a straight line through the origin, bisecting the first and third quadrants. Graph of this function confirms this unique property.

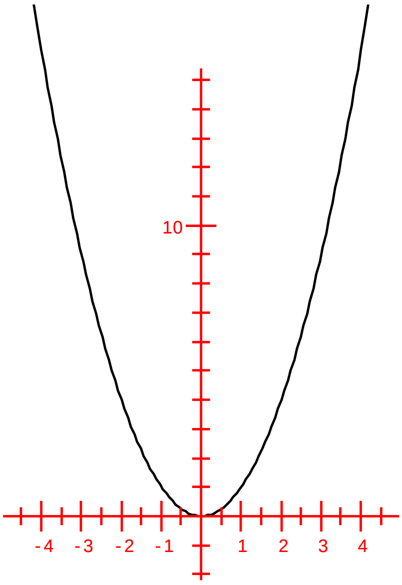


Properties:

1. Domain and range are all real numbers.
2. Continuous and smooth.
3. Always increasing. Hence it is a one-to-one function.
4. Symmetric with respect to the origin. This function is an odd function.
5. Not bounded.
6. No local extrema.
7. No asymptotes.
8. Intersects the axes only at the origin.

The Square Function:

This function is also referred to as "the squaring function" because it maps each real number to its own square. Algebraic notation is $f(x) = x^2$. The shape of the graph is a parabola. This is a "U" shape. and the graphical representation is shown below:

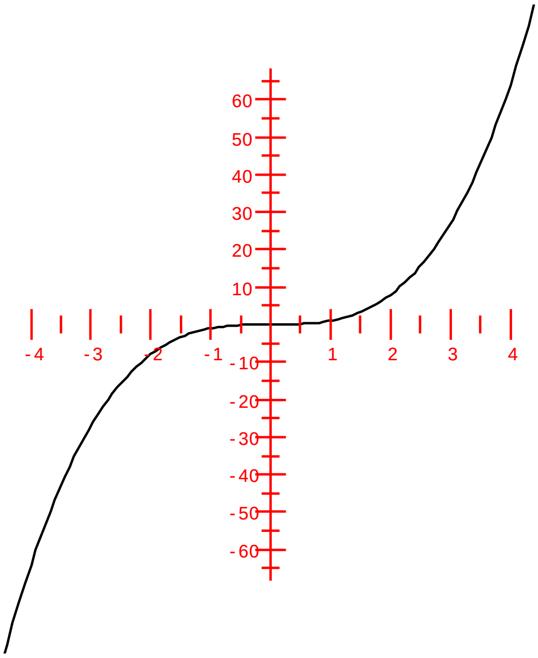


Properties:

1. Domain is all real numbers and range is all non-negative real numbers.
2. Continuous and smooth.
3. Decreases in the interval $(-\infty, 0)$ and then increases in the interval $(0, \infty)$. Hence, this is not a one-to-one function.
4. Symmetric with respect to the y-axis. Even function.
5. Bounded below by the point $(0, 0)$ but unbounded above.
6. Local minimum at $(0, 0)$.
7. No asymptotes.
8. This function has a reflection property, that is very useful in making flashlights, satellite dishes,etc.
9. Intersects the axes only at the origin.

The Cubic Function:

This function maps each real number to its cube. Hence it is also sometimes referred to as "the cubing function." Algebraic notation is $f(x) = x^3$. The graph passes through the origin, and though the first and third quadrants:

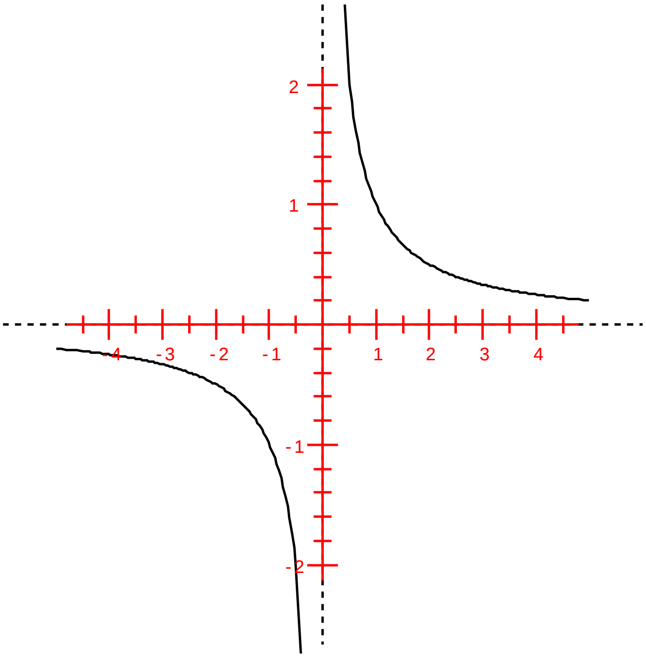


Properties:

1. Domain and range are all real numbers.
2. Continuous and smooth.
3. Always increases. This is a one-to-one function.
4. Symmetric with respect to origin. Odd function.
5. Unbounded both above and below.
6. No local extrema. But this function has a point of inflection at the origin $(0, 0)$ as the curvature of the graph changes at this point.
7. No asymptotes.
8. Intersects the axes only at the origin.

The Reciprocal Function:

This function maps each non-zero real number to its reciprocal. Algebraic notation is $f(x) = \frac{1}{x}$. Graph of this function is called a hyperbola. It is shown below:

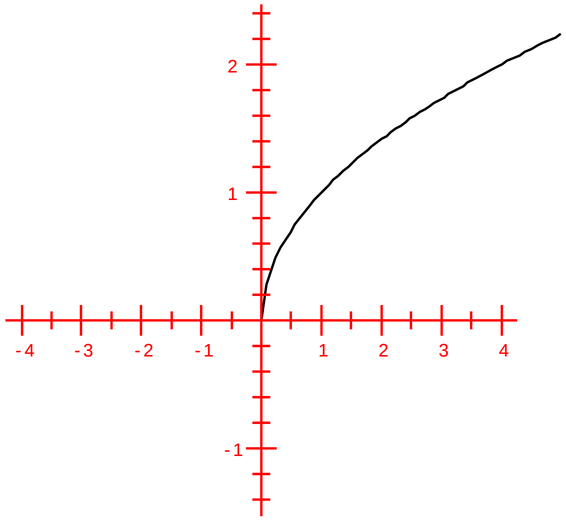


Properties:

1. Domain is all non-zero real numbers and range is also all non-zero real numbers.
2. This is a smooth curve, but it has a discontinuity at the origin $(0, 0)$.
3. The function decreases in the third quadrant in the interval $(_\infty, 0)$,and increases in the first quadrant in the interval $(0, \infty)$.
4. Symmetric with respect to the origin. Odd function.
5. Unbounded both below and above.
6. No local extrema.
7. For the graph of this function, y-axis is the vertical asymptote and x-axis is the horizontal asymptote.
8. No intercepts. That is, the graph does not intersect the coordinate axes.
9. This function also has a reflection property that is useful in making many devices like satellite dishes, etc.

The Square Root Function:

This function maps each non-negative real number to its square root. Algebraic notation is $f(x) = \sqrt{x}$. The graph covers only the first quadrant, as shown by the graph below:

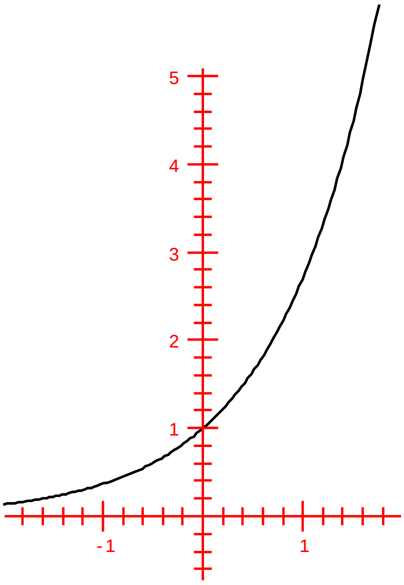


Properties:

1. Domain and range are all non-negative real numbers.
2. smooth and continuous curve in its domaain.
3. Graph always increases in its domain. It is a one-to-one function.
4. No symmetry.
5. Bounded below by the origin $(0, 0)$, but not bounded above.
6. The graph as an absolute minimum at $(0, 0)$. Other than this, no local extrema.
7. No asymptotes.
8. Intersects the axes only at the origin.

The Exponential Function:

Algebraic notation is $f(x) = e^x$. This function maps each real number to e^x. Algebraic notation is $f(x) = e^x$. The number e is an irrational number like π that has a variety of applications. Graph of this function is shown below:

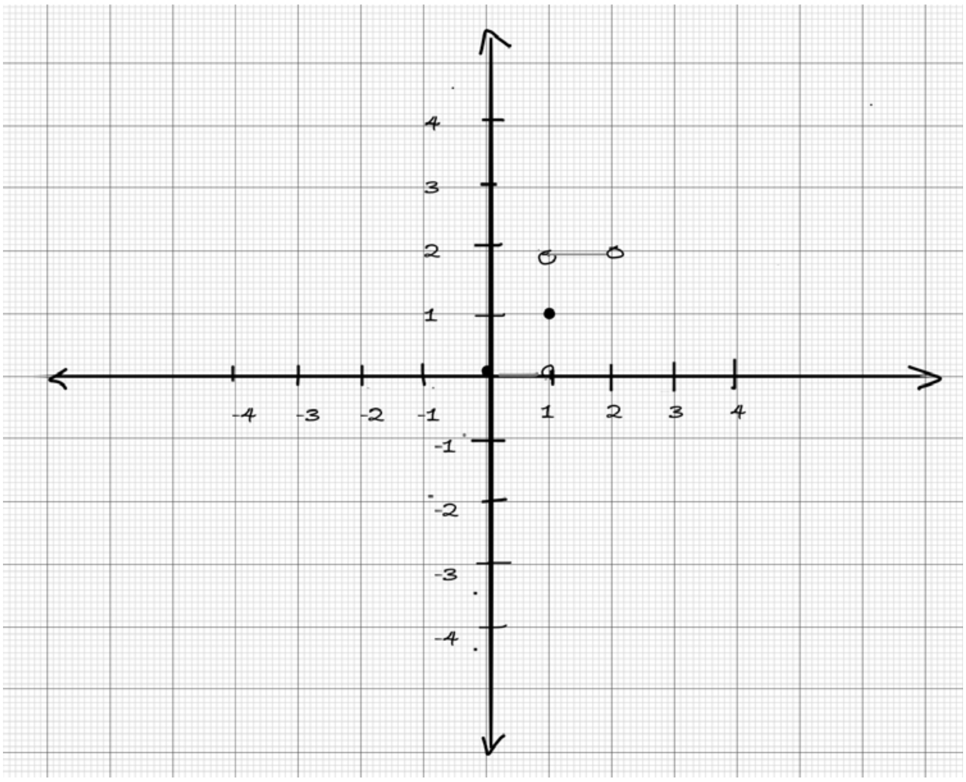


Properties:

1. Domain is the set of all real numbers $(-\infty, \infty)$. Range is the set of all positive real numbers $(0, \infty)$.
2. Smooth and continuous.
3. Graph always increases. It is a one-to-one function.
4. No symmetry.
5. Bounded below by the x-axis. Not bounded above.
6. No local extrema.
7. Horizontal asymptote is $y = 0$ (x-axis).
8. No x- intercept. y-intercept is $(0, 1)$.

The Natural Logarithm Function:

This function maps each positive real numbers to it natural logarithm. Algebraic notation is $f(x) = \ln x$. This is the inverse of the exponential function. The graph of the natural logarithm function is the reflection of the exponential function on the identity line, as is the characteristic of any pair of inverse functions. Graph of the function is shown below:



Properties:

1. Domain is all positive real numbers $(0, \infty)$. Range is all real numbers $(-\infty, \infty)$.
2. Smooth and continuous.
3. Always increases, but increases very slowly. One-to-one function
4. No symmetry.
5. Unbounded both above and below. But bounded on the left side by the y-axis.
6. No local extrema.
7. Vertical asymptote y-axis ($x = 0$).
8. No y-intercept. x-intercept is $(1, 0)$.

The Sine function:

This is one of the basic trigonometric function that maps each real number to its "sine" value. Algebraic notation is $f(x) = \sin x$. The wave- like shape of this curve has variety of applications. The graph is shown below:

Properties:

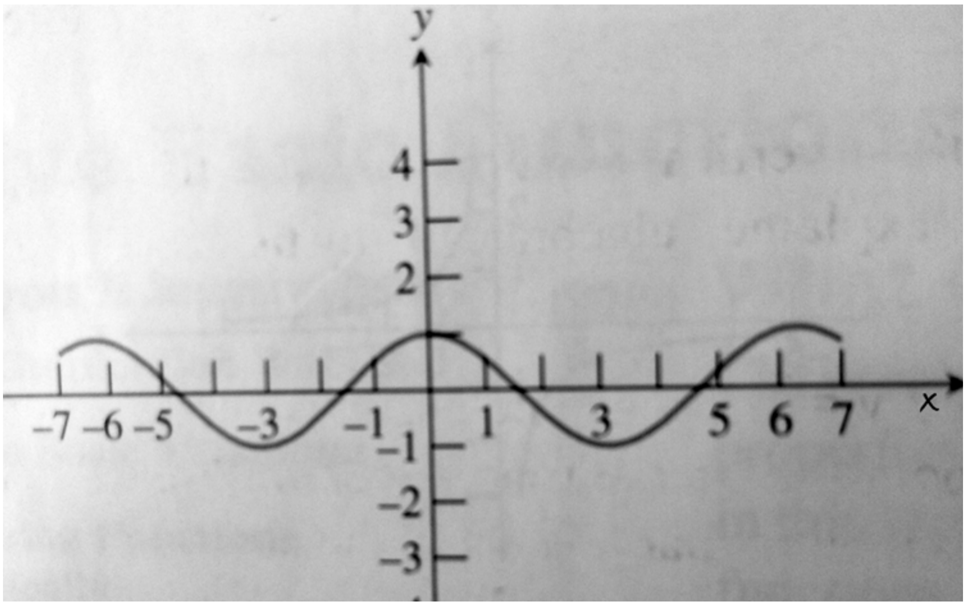
1. Domain is all real numbers $(-\infty, \infty)$. Range is between -1 and 1

, $[-1, 1]$.

2. Smooth and continuous.
3. It has wave-like shape, extending on both sides. Increases and decreases alternately. Not a one-to-one function unless a restricted domain is specified.
4. Symmetric with respect to the origin. This is an odd function.
5. Bounded below by $y = -1$ and bounded above by $y = 1$.
6. Local extrema at all odd-multiples of $\frac{\pi}{2}$. That is, at the points where $x = \frac{(2n+1)\pi}{2}$, where n is any integer. Maxima and minima alternate, with $x = \frac{\pi}{2}$ being a local maxima.
7. The graph has x-intercepts at the points where $x = n\pi$, where n is any integer. It intersects y-axis at the origin.
8. No asymptotes.
9. This is a periodic curve, with period 2π.That is the graph repeats itself in the interval of every 2π.

The Cosine Function:

This is another basic trigonometric function, that maps each real number to the "cosine" value of the numbers. Algebraic notation is $f(x) = \cos x$. The graph of this function also has wave-like shape.Graph is shown below:



Properties:

1. Domain is all real numbers $(-\infty, \infty)$. Range is between -1 and 1, $[-1, 1]$.

2. Smooth and continuous.
3. It has wave-like shape, extending on both sides. Increases and decreases alternately. Not a one-to-one function unless a restricted domain is specified.
4. Symmetric with respect the y-axis. This is an even function.
5. Bounded below by $y = -1$ and bounded above by $y = 1$.
6. Local extrema at all multiples of π. That is, at the points where $x = n\pi$, where n is any integer. Maxima and minima alternate, with $x = 0$ being a local maxima.
7. The graph has x-intercepts at the points where $x = n\pi$, where n is any integer. It intersects y-axis at the origin.
8. No asymptotes.
9. This is a periodic curve, with period 2π.That is the graph repeats itself in the interval of every 2π.

The Absolute Value Function

This function maps each real number into the 'absolute value' of that number. Algebraic notation is $f(x) = |x|$. Graph is shown below:

Properties:

1. Domain is all real numbers: $(-\infty, \infty)$. Range is all non-negative real numbers: $[0, \infty)$.
2. Continuous, but not smooth. Graph has a sharp corner at the point $(0, 0)$.
3. The graph has a "V" shape, with the lowest point origin.
4. Symmetric with respect to the y-axis. Even function.
5. Local minimum at the origin $(0, 0)$. No local maxima.
6. No intercepts other than the origin.
7. No asymptotes.

The Greatest Integer Function:

This function maps each real number to the greatest integer less than that number. For example, the number 1.002 is mapped to 1, and the number 1.992 is also mapped to 1. Algebraic notation is $f(x) = \int(x)$. Functions that have similar graphs are called **Step Functions.** Graph is shown below:

Properties:

1. Domain all real numbers $(-\infty, \infty)$. Range is all integers.
2. Neither smooth, nor continuous.
3. The graph is made up of small horizontal segments. It has jump-discontinuity at each integer.
4. Symmetric with respect to the origin. Odd function.
5. No local extrema.
6. Intersects the x-axis in the interval $[0, 1)$, and the y-axis at the origin.
7. No asymptotes.

The Logistic Function:

This function has many applications in biology, business, etc. Algebraic notation is $f(x) = \frac{1}{1+e^{-x}}$. Graph of this function is shown below:

Properties:

1. Domain is all real numbers: $(-\infty, \infty)$. Range is between $y = 0$ and $y = 1$. That is, in the interval $(0, 1)$.
2. Smooth and continuous.
3. No symmetry.
4. No x- intercept. Intersects y- axis at the point $(0, 0.5)$.
5. No local extrema.
6. There are two horizontal asymptotes: the x- axis and the line $y = 1$.

Apart from these basic functions, another type of function that is of importance in Calculus is **Piece-wise Function**.

A piecewise defined function is a function which has different expressions on the rights, for different intervals. That is, the real number set is subdivided into two or more intervals, and the function behaves differently in each of these intervals. Examples of such functions are provided below:

EXAMPLE 1:

$$y = \begin{cases} x^2, & x < 0 \\ 0, & x = 0 \\ x + 1, & x > 0 \end{cases}$$

This function is a parabole for the domain $(-\infty, 0)$. At $x = 0$, $f(x)$ is also 0. Then it is straight line for the interval $((0, \infty)$. The graph confirms that there is a jump discontinuity at $x = 0$.

EXAMPLE 2:

$$f(x) = \begin{cases} 0, & 0 \leq x < 1 \\ 1, & x = 1 \\ 2, & 1 < x < 2 \end{cases}$$

This function graphs as a small horizontal line segment in the interval $[0, 1)$, a point at $(1, 1)$, and another small horizontal line segment in the interval $(1, 2)$. This is also has a jump discontinuity at $x = 1$.

EXAMPLE 3:

$$f(x) = \begin{cases} x^2, & x \leq 0 \\ \sqrt{x} & x > 0 \end{cases}$$

This function is a square function for the interval $(-\infty, 0]$ and a square root function on the interval $(0, \infty)$.

There are no practice problems for this module. These twelve basic functions will be used to produce more functions using transformations and practice problems will be provided in the next module.