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Chapter P - Preliminary Concepts

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P10: Problem Solving using Algebra

Word problems, which are also referered to as 'application problems', describe relationships among quantities. These quantities contain numbers with appropriate units. The most general process of problem solving involves the following steps:

1. Read carefully and understand the problem.
2. Represent the given information in a diagram, if needed.
3. Identify the given information, and what you need to determine.
4. Identify appropriate relationship among the quantities involved. Use a chart or a table if needed.
5. Reread the problem. Write an equation or an inequality that represents the relationships among the numbers in the problem.
6. Check the units and use conversions if needed to ensure all relevant quantities are of the same unit.
7. Solve the equation or inequality and find the required quantities.
8. Interpret what you found with respect to the problem and state your final answer, with appropriate unit.

The major aspect of problem solving is the ability to translate the given relationship into mathematical sentences. Below are the basic operations in mathematics and the words/phrases that would translate into these operations:

1. $(+)$ $\Rightarrow$ addition of, sum, more than, increased by, added to, etc.
2. $(-)$ $\Rightarrow$ difference between, decreased by, less than, subtracted

from, etc.

3. $(\times) \quad \Rightarrow \quad$ times, product of, of, multiplied by, etc.
4. $(\div) \quad \Rightarrow \quad$ per, divided by, quotient of, ratio of, etc.

Apart from the above operations, the following list gives translations of phrases into equality or inequalities:

1. $(=) \quad \Rightarrow \quad$ is, equals, results in, becomes, etc.
2. $(<) \quad \Rightarrow \quad$ less than. When this phrase occurs in the problem, write the quantity given after "than" first, then write the sign $(-)$, and after that write the quantity given before this phrase or after the word "by".
3. $(>) \quad \Rightarrow \quad$ greater than. When this phrase occurs in the problem, write the quantity given after "than" first, then write the sign $(+)$, and after that write the quantity given before this phrase or after the word "by".
4. $(\leq) \quad \Rightarrow \quad$ at most, not greater than, not more than, maximum. When the phrases that end with "than" occur in the problem, write the quantity given after "than" first, then write the sign $\leq$, and after that write the quantity given before this phrase.
   $(\geq) \quad \Rightarrow \quad$ at least, not less than, minimum. When the phrases that end with "than" occur in the problem, write the quantity given after "than" first, then write the sign $\geq$, and after that write the quantity given before this phrase.

The above translations just provide quidance for solving application problems. In addition, we need to use logic and common sense also, to translate the given information into mathematical sentences.

Below are some examples of basic word problems:

Example 1:

Write the sentence "10 less than 41" as a mathematical sentense:

Solution: $41 - 10$

Example 2:

Write the statment "The sum of -2 and 8 decreased by 3 as a mathematical sentence.

Solution: $(-2+8)-3$

Example 3:

Write the statement "A number is greater than the product of 20 and 3 by 6

Solution: $x=(20\times 3)+6$

Example 4:

Find a number that equals the quotient of 35 and -5 increased by the product of -2 and 6.

Solution: Let the required number be x.

$$\begin{aligned} x &= 35 \div -5 + (-2\times 6) \\ x &= -7 + (-12) \\ x &= -19 \end{aligned}$$

Example 5:

A personal chef is preparing dinner for her client who is on a 1200 calories per-day restricted diet. If the client has already consumed 745 calories, mostly consisting of foods high in protein, how many calories can this client have for dinner?

Solution: Maximum calories intake of the client is 1200. She has already consumed 745 calories.
Let the amount of calories the client can have for dinner be x
Then, the eqution for this problem is:

$$x = 1200 - 745$$
$$x = 455$$

So, the answer is " The client can have at most 445 calories for dinner."

EXAMPLE 6:

Corey has two coupons for his local store. One coupon is for $\$5$ off his total purchase and the other is for 20% off a single item. If he can only use one coupon to purchase a $\$16$ mirror and a $\$5$ set of pillow cases, answer the following questions:
a. Which coupon will save him the most money?
b. How much will he save?

Solution: Corey's total purchase is $\$21$.
Let C_1 be the amount he needs to pay if he uses the first coupon.
Then,

$$C_1 = 21 - 5$$
$$C_1 = 16$$

This means he has to pay $\$16$ is he uses the first coupon.
Let C_2 be the amount he needs to pay if he uses the second coupon.
This means he can have 20% off on any single item. Natually, he would choose the mirror, which costs more.
Then,

$$C_2 = 16 \times 0.80 + 5$$
$$C_2 = 12.80 + 5$$
$$C_2 = 17.80$$

This means Corey has to pay $\$17.80$ with this second option.

Conclusion: a. Corey would use the first coupon; b. He would save $\$1.80$.

Example 7:

A newlywed couple wants to rent a taxicab from their hotel to downtown Chicago. If a taxicab charges $\$3$ upon a customer's entering the taxi, then $\$0.25$ for each mile traveled,
(a) how much will the couple pay for a 12-mile taxi ride?
(b) The cab driver earned $\$50$ on a certain day for a ride. How far did he drive on that ride?

Solution:
(a) Let the amount the couple pay be $\$x$.
Then,

$$\begin{aligned} x &= 3 + 12 \times 0.25 \\ x &= 3 + 3 \\ x &= 6 \end{aligned}$$

Conclusion: The couple will have to pay $\$6$ for the ride.

(b) Let x represent the number of miles the driver drove for that ride. Then,

$$\begin{aligned} 50 &= 3 + 0.25 \times x \\ 50 - 3 &= 0.25x \\ 47 &= 0.25x \\ x &= 47 \div 0.25 \\ x &= 188 \end{aligned}$$

Conclusion: The cab drive drove 188 miles during that ride.

Example 8:

A carpenter has a board that is 18 meters long. He needs to cut it into two pieces so that one piece is half as long as the other. What will be the length of each piece?

Solution: Let the length of the longer piece be x meters.
Then, the length of the shorter piece would be $18 - x$ meters.
As per the given information,

$$\begin{aligned} 18 - x &= \frac{1}{2} \times x \\ 2 \times (18 - x) &= x \\ 36 - 2x &= x \\ 36 &= 3x \\ 12 &= x \end{aligned}$$

Conclusion: One piece is 12 meters long. So, the shorter piece is (18-12) 6 meters long. Verify that the second piece is half the length of the first piece, which satisfies the given information.
Alternately, if we assume the length of the shorter piece as x meters, then the length of the longer piece would be $2x$. In this case, the equation would become $18 - x = 2x$. Solving, we get $x = 6$, which is the length of the shorter piece.

Example 9:

The table below shows percentages of female and male population employed in the United States during a 8 year period.

Year	*Female*	*Male*
1	32.5	83.5
2	35.1	82.3
3	36.9	80.9
4	41.1	81.1
5	42.8	77.9
6	47.7	76.5
7	50.1	73.2
8	54.9	74.5

a. According to the above model, what has been the trend in females joining work force from year 1 through year 8?
b. In what 4-year interval year 1 - year 4 or year 5 - year 8, did the percentage of women who were employed change the most?
c. Model the data algebraically with linear equaions of the form $y = mx + c$. Write one equation for women employment and another equation for male employment.
d. If the percentages continue to follow the linear models you found in (c), what will be the employment percentages for women and men in the year 11?

Solution:

(a) Notice that the employment percentages of women was continuously increasing in this period. That average percentage increase is given by the slope of the graph when we plot number of year on x- axis and percentage of female population on y-axis. So, the average percentage increase would be:

$$(54.9 - 32.5) \div (8 - 1) = 3.2$$

Conclusion: The percent of women workforce was increasing at an average rate of 3.2% per year during this period.

(b) For this, we shall find the average increase in each of the 4-year periods:
For year 1 - year 4, average percentage increase is

$$(41.1 - 32.5) \div (4 - 1) = 2.87$$

For year 5 - year 8, average percentage increase is

$$(54.9 - 42.8) \div (8 - 5) = 4.01$$

Conclusion: Percentage of women who were employed changed the most during the period of year 5 - year 8.

(c) For women: The slope already calculated in part (a), which is 3.2. Taking (x_1, y_1) as $(3, 36.9)$, we have the linear model

$$\begin{aligned} y - 36.9 &= 3.2(x-3) \\ y - 36.9 &= 3.2x - 9.6 \\ y &= 3.2x + 27.3 \end{aligned}$$

For men: Slope is $m = (74.5 - 83.5) \div (8-1) = -1.3$
So, taking the point (x_1, y_1) as $(3, 80.9)$, the model for percentage change men who are employed during this period is

$$\begin{aligned} y - 80.9 &= -1.3(x-3) \\ y - 80.9 &= -1.3x + 3.9 \\ y &= -1.3x + 84.8 \end{aligned}$$

Conclusion Equation for women is $y = 3.2x + 27.3$ and equation for men is $y = -1.3x + 84.8$

(d) Employment percentage of women in year 11:

$$\begin{aligned} y &= 3.2 \times 11 + 27.3 \\ y &= 62.5 \end{aligned}$$

Employment percentage of men in year 11:

$$\begin{aligned} y &= -1.3 \times 11 + 84.8 \\ y &= 70.5 \end{aligned}$$

Conclusion: In year 11, the women in work force would be 62.5% and the men in work force would be 70.5%

Practice Problems

(1) The sum of three consecutive integers is 126. Find the integers.

(2) To rent a car for 1 day, a rental car company charges $\$19.95$ plus $\$0.26$ per mile to rent one of its economy cars. How much would it cost to rent a car for 1 day if you intend to drive 68 miles?

(3) Twice the sum of two consecutive integers is 18 more than five times the largest of the two integers. Find the integers.

(4) Amherst apartments purchased a $\$50,000$ building that depreciates at $\$2000$ per year over a period of 25 years.
(a) Write a linear equation giving the value y of the building in terms of the years x after the purchase;
(b)In how many years will the value of the building be $\$24,400$?

(5) Tickets to a theater were $\$19$ for seats near the front and $\$14$ for rear seats. There were 525 more rear seats sold than front seats, and sales for all tickets totaled $\$31,770$. How many of each kind of ticket were sold?

(6) There were two pieces of wire of equal length. One piece was shaped into a square and the second piece was shaped into an isosceles triangle. The base of the isosceles triangle is 4 cm shorter than a side of the square, and each leg is 9 cm longer than a side of the square. How long was each piece of wire?

(7) Amy has $\$8$ less than Maria. Together, they have $\$30$. How much money does each girl have?

(8) To use a certain computer data base, the charge is $\$30$/h during the day and $\$10.50$/h at night. If a researcher paid $\$411$ for 28 hours of use, find the number of hours charged at the daytime rate and at the nighttime rate.

(9) In a certain factory, the cost C, in thousands of dollars, of producing x tons of material is given by the equation $C = 0.3x + 3.5$. The revenue R, in thousands of dollars, from selling x tons of that material is given by the equation $R = 0.5x$. Find the amount of material to be produced for break-even. ($C = R$).

(10) Find the coordinates of a point one-thirds of the distance from $a = -3$ to $b = 9$ on the number line.

(11) Americans' personal income, in trilions of dollars, during the years 1998 to 2003 is given in the following table:

$year$	$Amount$
1998	7.4
1999	7.8
2000	8.4
2001	8.7
2002	8.9
2003	9.2

(a) Write a linear equation for Americans' income y in terms of the year x using the data for 1998 and 1999.
(b) Use the equation to predict Amercans' income in 2006.

(12) An airplane climbs at takeoff with slope $m = \frac{3}{8}$. How far in the horizontal direction will the airplane fly to reach an altitude of 12,000 ft above the takeoff point?

(13) A projectile is launched straight up from ground level with an initial velocity of 256 ft/sec.
(a) When will the projectile's height above the ground be at least 768 feet?

(14) For a certain gas, $P = 400 \div V$ where P is the pressure and V is the volume. If $20 \leq V \leq 40$, what is the corresponding range for P?

$10 \leq P \leq 20$

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