Dr. Lalitha Subramanian

About the Author

Aksharam

PRECALCULUS

Chapter P - Preliminary Concepts

PDF Version

P1: Real Numbers

The set of real numbers includes the following subsets:

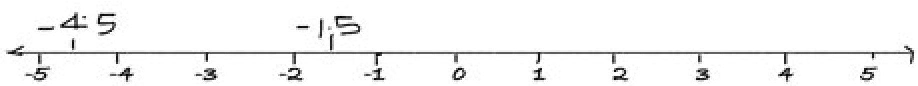
1. Set of digits $D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. This set has exactly 10 elements.
2. Set of natural numbers $N = \{1, 2, 3, 4, \ldots\ldots\ldots\}$. This set has lower boundary 1 but no upper boundary.
3. Set of whole numbers $W = \{0, 1, 2, 3, 4, \ldots\ldots\}$. This set has lower boundary 0 ,but no upper boundary
4. Set of integers $Z = \{\ldots\ldots\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4......\}$ This set has no lower or upper boundary.
5. Set of rational numbers Ra = {All numbers that can be written as $\frac{p}{q}$, $p.q \in Z$ and $q \neq 0$ }. This set is also not bounded, and contains all numbers that can be written as terminating or recurring decimals.
6. Set of irrational numbers $IRa = \{x/x$ cannot be written as $\frac{p}{q}\}$.
   This contains all numbers that cannot be writtenas either terminating or recurring decimals.
7. Set of real numbers $R = \{x/x$ is either rational or irrational}. This means that any real number can be written as a decimal.

The set of real numbers is ordered. This means that any two real numbers are compared using any one of the trichotomy "less than", equal to", or "greater than".

The four symbols used to compare any two real numbers are $<$, $\leq$, $>$, and $\geq$.

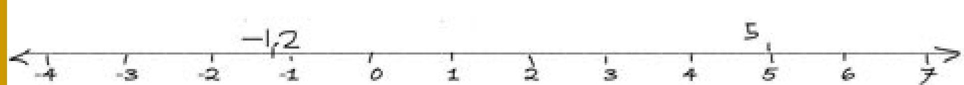
EXAMPLE 1:

$-4.5 < -1.5$. We see this on the number line.



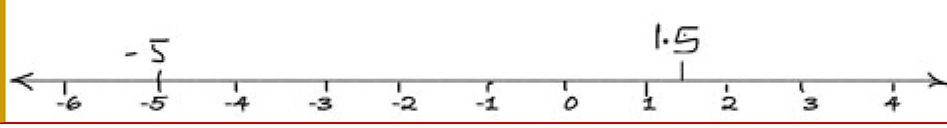
EXAMPLE 2:

$5 > -1.2$. We see this on the number line.



EXAMPLE 3:

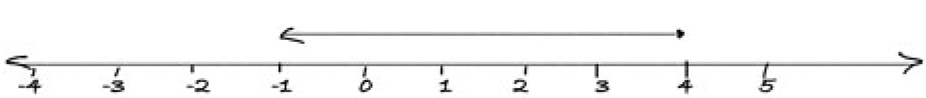
$-5 < 1.5$. This is shown on the number line.



Basically, if a and b are positive real numbers and a < b , then, $-a$ > $-b$.

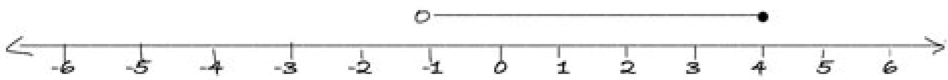
EXAMPLE 4:

$x \leq 4$ means x can be any real number less than or equal to 4. This is shown on the number line as



EXAMPLE 5:

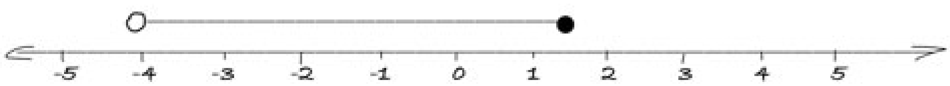
$-1 < x \leq 4$ represents a real number x that is greater than - 1 , but less than or equal to 4 . On the number line, this is graphed as



Inequalities define intervals on the real number line. Intervals can be bounded on both ends, only one end, or with no bound on either side.

EXAMPLE 6:

$-4 < x \leq 1.5$ is written as an interval as $(-4, 1.5)$. This can be shown on the number line as



There are four types of bounded intervals and four types unbounded intervals. The four types of bounded intervals are:

1. (a, b) represents all real numbers x such that $a < x < b$
2. $[a, b]$ represents all real numbers x such that a $\leq$ x $\leq$ b
3. $(a, b]$ represents all real numbers x such that a $< x \leq b$
4. $[a, b)$ represents all real numbers x such that $a \leq x < b$

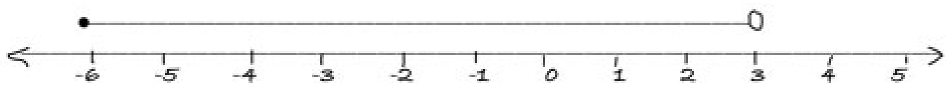
The four types of unbounded intervals are:

1. $[a, \infty)$ represents all real numbers greater than or equal to a . This interval has lower bound a, but no upper bound
2. (a, ∞) represent all real numbers greater than a . This interval has a lower bound a , though a is not included in the inteval. It has no upper bound.

3. $(-\infty, b]$ represents all real numbers less than b . this interval has no lower bound,but has upper bound b .
4. $(-\infty, b)$ represents all real number less than, but not equal to b . This interval has no lower bound but upper bound b, though b is not included in the interval.

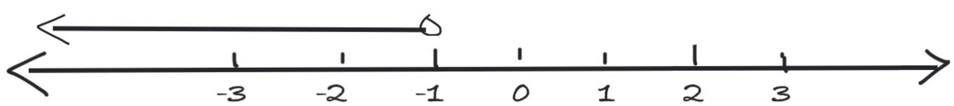
EXAMPLE 7:

$[-6, 3)$ is written as inequality $-6 \leq x < 3$.



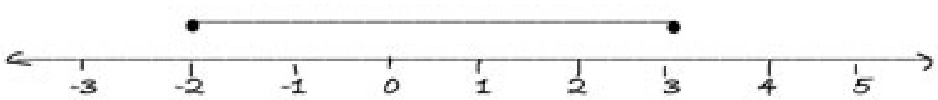
EXAMPLE 8:

$(-\infty, -1)$ is written as the inequality $-\infty < x < -1$.



EXAMPLE 9:

The inequality $-2 \leq x \leq 3$ is written as $[-2, 3]$ as interval.



Algebra involves the use of letters and symbols to represent real numbers. A symbol (a letter or any other symbol) represents an unspecified number whereas a constant is a symbol (a letter, any other symbol, of a number) represents a specific real number. An algebraic expression is a combination of variables and constants connected using addition, subtraction,multiplication, division, exponents, logarithm, etc. Of these operations, addition and subtraction are opposite operations, multiplication and division are opposite operations,and exponents and logarithm are oppositie operations.

Basic properties of the first four operations are:

1. **Commutative property:** For any two real numbers, addition and multiplication are commutative.
   That is, For two real numbers u and v , $u+v=v+u$ and $uv=vu$;
   But subtraction and division are not commutative. That is, $u-v\neq v-u$ and $\frac{u}{v}\neq\frac{v}{u}$.
2. **Associative property:** For any three real numbers, addition and multiplication are associative. That is,for real numbers u , v , and w , $(u+v)+w=u+(v+w)$ and $(uv)w=u(vw)$
3. **Distributive property:** For any three real numbers, multiplication is distributive over addition. This means, $u(v+w)=uv+uw$ and $(u+v)w=uw+vw$.
4. **Identity property:** For any real number, additive identity is 0 and multiplicative identity is 1 . This means, for a real number u
   $u+0=0+u=u$ and $u*1=1*u=u$.
5. **Inverse property:** When two real numbers are added, if the result is the additive identity 0 , then these two numbers are called additive inverses of opposites. When two real numbers multiplied and the result is the multiplicative identity 1 ,then these two numbers are called multiplicative inverses or reciprocals.

Another important skill students should master is the use of laws of exponents. These laws are listed below:

1. $a^m\times a^n=a^{(m+n)}$.
2. $\frac{a^m}{a^n}=a^{(m-n)}$.
3. $(ab)^m=a^mb^m$

4. $(\frac{a}{b})^m = \frac{a^m}{b^m}$.
5. $(u^m)^n = u^{mn}$.
6. $u^0 = 1$
7. $u^1 = u$

Some examples of simplifying expressions involving powers are provided below:

Example 10:

$$2ab^3(5a^2b^5) = 10a^3b^8$$

Example 11:

$$\frac{u^2v^{-2}}{u^{-1}v^3} = \frac{u^3}{v^5}$$

Example 12:

$$(\frac{x^2}{2})^{-3} = \frac{8}{x^6}$$

Example 13:

$$[(2x^3y)^3](4x^2y^2) = (8x^9y^3)(4x^2y^2) = 32x^{11}y^5$$

Example 14:

$$\left(\frac{2}{3}xy\right)\left(\frac{-9}{10}x^2y^2\right) = \frac{-3}{5}x^3y^3$$

Example 15:

Simplify $\sqrt[3]{\frac{a^5b^3}{c^2}}$

Solution:

$$\left[\sqrt[3]{\frac{a^5b^3}{c^2}}\right] = \left[\left(\frac{a^5b^3}{c^2}\right)^{1/3}\right] = \left[\frac{a^{5/3}b^{3/3}}{c^{2/3}}\right] = \left[a^{5/3}bc^{-2/3}\right]$$

Practice Problems (click on the problem to view the answer.)

(1) Write the inequality $x > -3$ in interval notation.

(2) Write the inequality $-2 \leq x < 5$ in interval notation.

(3) Write the interval $(-\infty, 7)$ as inequality.

(4) Write the interval $[-3, 3]$ as inequality.

(5) Write the statement
"x is greater than -3 and less than or equal to 4 as an interval and as an inequality.

(6) Use words to describe the inequality $4 < x \leq 9$.

(7) Use words to describe the interval [- 3, ∞).

(8) Convert the interval (- 3, 4] to inequality notation. Find the end points and state whether the interval is bounded or unbounded.

(9) Describe the statement "No item in John's store costs more than $5.00 as an interval and as an inequality.

(10) Describe the statement " The price of a gallon of gasoline varies from 1.099 to $3.199 as an interval and as inequality.

(11) Expand the expression $a(x^2 + b)$.

(12) Factor the expression $a^3 z + a^3 w$.

(13)Simplify the expression $\frac{x^4 y^3}{x^2 y^5}$.

(14) Simplify the expression $\frac{(3x^2)^2 y^4}{3y^2}$.

(15) Simplify the expression $\left(\frac{4}{x^2}\right)^2$

(16) Simplify the expression $\left(\frac{2}{xy}\right)^{-3}$

(17) Simplify the expression $\frac{(x^{-3}y^2)^{-4}}{(y^6x^{-4})^{-2}}$

(18) Simplify the expression $\left(\frac{4a^3b}{a^2b^3}\right)\left(\frac{3b^2}{a^2b^4}\right)$

(19) Simplify the expression $\frac{3\times 7^2\times 11^8}{21\times 11^3}$

(20) Simplify the expression $\frac{2^3\times 3^4\times 3}{3\times 32}$

(21) Simplify the expression $\frac{4^5\times a^8b^3}{4^5\times a^5b^2}$

(22) Simplify the expression $\frac{4x^3y^2}{2xy^2}$

(23) Simplify the expression $\left(\frac{9v^2}{16w^4}\right)^{\frac{1}{2}}$

(24) Simplify the expression $\sqrt{\frac{x^{-2}y^2}{25x^4}}$

(25) Simplify the expression $\frac{6x^2y^{-2}}{\sqrt[3]{8x^{-3}}}$